A ReaDiNet workshop on deterministic and stochastic PDEs

24-27 Nov 2025
Obernai
France

Table of contents

Optimal rate of convergence to fast diffusion on bounded domains

Goro Akagi,
Tohoku University, Japan
goro.akagi@tohoku.ac.jp

Abstract

In this talk, we overview recent developments for quantitative analysis of asymptotic behavior of energy solutions to the Cauchy–Dirichlet problem for fast diffusion equations posed in bounded domains. It is well known that every energy solution vanishes in finite time and a suitably rescaled solution converges to an asymptotic profile, which is a nontrivial solution for a semilinear elliptic equation. Bonforte and Figalli (CPAM, 2021) first determined an exponential rate of convergence to nondegenerate positive asymptotic profiles for nonnegative rescaled solutions in a weighted L^2 norm for smooth (at least C^2) bounded domains by developing the so-called nonlinear entropy method. On the other hand, the speaker (ARMA, 2023) developed an energy method along with a quantitative gradient inequality and also proved the same exponential convergence in the Sobolev norm for bounded $C^{1,1}$ domains. The optimality of the exponential rate was conjectured in view of some formal linearized analysis; however, it was not proved due to some difficulty arising from nontrivial stability nature of asymptotic profiles in the fast diffusion setting. Furthermore, the nondegeneracy of asymptotic profiles was indispensable in these works. In this talk, these results are extended to possibly sign-changing (nondegenerate) asymptotic profiles as well as general bounded domains by improving the energy method as well as quantitative gradient inequality. Moreover, the optimality of the exponential rate is also proved. Furthermore, such a quantitative analysis is also extended to degenerate asymptotic profiles. This talk is based on recent joint works with Yasunori Maekawa (Kyoto University) and Norihisa Ikoma (Keio University).

Study of the influence of the cell nucleus on cell motility: modelling and numerical simulations

Claire ALAMICHEL

LMJL, École Centrale de Nantes, Nantes Université,
claire.alamichel@ec-nantes.fr

The cell is a complex system containing several molecules that interact with each other and with the external environment, leading to various remarkable phenomena such as cell movement. Cell motility is a phenomenon involved in many biological processes such as cancer spread, immune response, wound healing, or embryonic development.

After presenting the biological context, I will present a 2D free-boundary model for this phenomenon taking into account the nucleus dynamic. The cell is modelled by a droplet of incompressible fluid containing a rigid structure modelling the nucleus. The dynamics of the free boundary depend on surface tension, on the force induced by the nucleus on the cell, and on coupling with polarity markers present in the fluid. I will present results on the existence and stability of steady states. Finally, I will present a finite element numerical scheme enabling numerical simulations to be carried out, highlighting the influence of the nucleus on cell motility.

This work is the result of collaborations with Nicolas Meunier and Raphaël Voituriez.

References

[1] C. Alamichel. *Study of the influence of the cell nucleus on cell motility*. Ph.D. thesis, Université Paris-Saclay ; Politecnico di Torino, 2024. 2024UPASM023.

[2] I. Lavi, N. Meunier, R. Voituriez, J., Casademunt, *Motility and morphodynamics of confined cells*, Phys. Rev. E. , 110, 078102 (2020).

On the field-road diffusion model

Matthieu Alfaro (Univ. Rouen Normandie)

The field-road model was introduced by Berestycki, Roquejoffre, and Rossi in 2013 to describe the spread of species or epidemics in the presence of rapid diffusion lines. In this talk, I will consider the purely diffusive field-road model as a starting point. I will discuss several aspects of it: an approximation using a system of particles, the fundamental solution, an entropy method to study the long-time behavior.

The talk is based on works implying C. Chainais-Hillairet, R. Ducasse, M. Mourragui and S. Tréton.

Branching particle systems and reaction-diffusion free boundary problems

Julien Berestycki [a],

[a] University of Oxford

Email: berestyc@stats.ox.ac.uk

The free boundary problem

$$\begin{cases} \partial_t u = \frac{1}{2}\Delta u + u, & t > 0,\ x > L_t, \\ u(t,x) = 0, & t > 0,\ x \leq L_t, \\ \int_{L_t}^{\infty} u(t,y)dy = 1, & t > 0, \\ u(t,x)dx \to u_0(dx) & \text{weakly as } t \to 0, \end{cases}$$

has long been conjectured to be in the universality class of the so-called FKPP reaction-diffusion equation. It appears naturally as the hydrodynamic limit of a branching-selection particle system, the N-BBM.

In this talk I will discuss recent results that confirms this. In particular I will discuss how the initial condition influences the fron position, the convergence to traveling waves; focusing in particular on the necessary and sufficient conditions under which we have convergence to the minimal travelling wave, and L_t has the Bramson asymptotics

$$L_t = \sqrt{2}t - \frac{3}{2\sqrt{2}}\log t + c + o(1) \quad \text{as } t \to \infty,$$

One of the main tool is a Brunet-Derrida relation between the initial condition and an integral transform of $(L_t)_{t\geq 0}$. Finally, I will discuss a more general free boundary problem that depends on a parameter β, where we see a transition from *pulled* to *pushed* behaviour (with *pushmi-pullyu* behaviour at the critical value of β).

Based on joint works with Sarah Penington and Oliver Tough.

Qualitative properties of the spreading speed of a population structured in space and in phenotype

Nathanaël Boutillon

We consider a nonlocal Fisher-KPP equation that models a population structured in space and in phenotype. The population lives in a heterogeneous periodic environment: the diffusion coefficient, the mutation coefficient and the fitness of an individual may depend on its spatial position and on its phenotype. The population spreads at some finite speed given by a Freidlin-Gärtner formula. We study the behaviour of the spreading speed in different scaling limits: small and large period, small and large mutation coefficient.

This amounts to studying the behaviour of the principal eigenvalue of an elliptic operator in which the second-order term is very small in one direction. We deal with the interaction between the slow and fast scales by using a probabilistic interpretation of the principal eigenfunction as a quasi-stationary distribution of some killed process, along with techniques from Hamilton-Jacobi equations.

References

[1] N. Boutillon. "Qualitative properties of the spreading speed of a population structured in space and in phenotype". In: *Journal de Mathématiques Pures et Appliquées* 204 (2025), p. 103804.

[2] N. Boutillon. "The principal eigenvalue problem for a strongly anisotropic second-order elliptic operator". In: *SIAM J. Math. Anal.* 57.4 (2025), pp. 3840–3877.

A variational approach to the semi-wave solutions of a free boundary problem with a multistable nonlinearity

Chueh-Hsin Chang
Department of Mathematics
National Chung Cheng University

Abstract

The semi-wave solutions is the traveling wave solutions defined in the half spaces which exhibit many important features for the spreading dynamics for the competing species. In this talk, we give the results about semi-wave solutions coming from the free boundary problems with monostable or bistable type reaction terms. We review the phase plane methods and provide variational methods for the existence of semi-wave solutions, and the dependence of wave speeds on the parameters in the Stefan conditions.

Diffusion-approximation and rate of convergence for the stochastic Zakharov system

A. de Bouard

CNRS, CMAP, Ecole polytechnique, Institut Polytechnique de Paris

We study he convergence of a Zakharov system, which couples a Schrödinger equation for the electric field envelope with a wave equation for the variation in ion density. This system is a simplified model for Langmuir turbulence and the introduction of a noise describes the influence of external perturbations of the ion density. In the subsonic limit, the system formally converges to a stochastic nonlinear Schrödinger equation, but this limit becomes singular in the presence of noise. It will be shown that the system can nevertheless be rewritten in the approximation-diffusion regime and that this limit can thus be studied rigorously using predictor-corrector methods. A rate of convergence is obtained in the linear case. This work is in collaboration with Grégoire Barrué, Arnaud Debussche and Rita Nader (ENS Rennes).

Global strong solutions for the triangular Shigesada-Kawasaki-Teramoto cross-diffusion system in three dimensions and parabolic regularisation for increasing functions

Hector Bouton * Laurent Desvillettes † Helge Dietert ‡

We prove the existence of global strong solutions to the triangular Shigesada-Kawasaki-Teramoto (SKT) cross-diffusion system with Lokta-Volterra reaction terms in three dimensions. A key part is the independent careful study of the parabolic equation $a\partial_t w - \Delta w = f$ with a rough coefficient a, homogeneous Neumann boundary conditions, and the special assumption $\partial_t w \geq 0$. By the same method, we obtain estimates for solutions to reaction-diffusion systems modelling reversible chemistry.

See the preprint at `https://arxiv.org/abs/2503.08186`.

*Email: `hector.bouton@ens.psl.eu`
Université Paris Cité and Sorbonne Université, CNRS, IMJ-PRG, F-75013 Paris, France
†Email: `desvillettes@imj-prg.fr`
Université Paris Cité and Sorbonne Université, CNRS and IUF, IMJ-PRG, F-75013 Paris, France.
‡Email: `helge.dietert@imj-prg.fr`
Université Paris Cité and Sorbonne Université, CNRS, IMJ-PRG, F-75013 Paris, France.

Models of mosquito population control strategies for fighting

Luis Almeida[1], **Yves Dumont**[2], **Michel Duprez**[3], **Yannick Privat**[4], **Nicolas Vauchelet**[5]

[1] Inria, Paris
[2] CIRAD, Montpellier
[3] Inria, Strasbourg
[4] IECL, Nancy
[5] LAGA, Paris

Résumé. In the fight against vector-borne arboviruses, an important strategy of control of epidemic consists in controlling the population of the vector, Aedes mosquitoes in this case. Among possible actions, a technique consist in releasing sterile mosquitoes to reduce the size of the population (Sterile Insect Technique). This talk is devoted to studying the issue of optimizing the dissemination protocol for each of these strategies, in order to get as close as possible to these objectives. Starting from a mathematical model describing the dynamic of a mosquitoes population, we will study the control problem and introduce the cost function standing for sterile insect technique. In a second step, we will consider a model with several patchs modeling the spatial repartition of the population. Then, we will establish some properties of these two optimal control problems. Finally, we will illustrate our results with numerical simulations.

Mots-clés : Optimal control, mosquitoes, ODE, PDE

Références

[1] *Sterile Insect Technique in a Two-Patch Model: Effects of Migration Rates on Optimal Control Strategies for Single-Patch Releases.* Y. Dumont, M. Duprez and Y.Privat. Submitted

[2] *Modeling the impact of rainfall and temperature on sterile insect control strategies in a Tropical environment.* Y. Dumont and M. Duprez. Journal of Biological Systems Vol. 32 (2024)

[3] *Optimal control strategies for the sterile mosquitoes technique.* L. Almeida, M. Duprez, Y. Privat and N. Vauchelet. J. Differential Equations 311 (2022), 229–266

ReaDiNet2025: A ReaDiNet Workshop on deterministic and stochastic PDEs

Hydrodynamic limit of a branching Markov process with selection

Brieuc Frénais
Institut Elie Cartan de Lorraine, Université de Lorraine

In this talk, we will focus on a certain particle system called the N-BMP (N-branching Markov process). This stochastic model involves particles moving on the real line according to a continuous-time Markov process, subject to two simultaneous mechanisms: each particle branches at rate 1, while the particle currently at the lowest location is removed.

We will study the hydrodynamic limit of this system, that the limiting distribution of particles as the population size at time 0 goes to infinity. This question has been studied for particles with Brownian trajectories since 2017, in relation with a free boundary problem for a reaction-diffusion PDE. We will present our extension of this convergence result to a wide class of Feller processes, and discuss the links with the corresponding free boundary problem.

This work is in collaboration with Jean Bérard (Institut de Recherche Mathématique Avancée, Université de Strasbourg).

Reika Fukuizumi (Waseda University)

Title : Analysis of a stochastic PDE in finite temperature quantum fluid dynamics

Abstract: We present recent developments in the mathematical study of the stochastic Gross-Pitaevskii equation. This talk is a summary of several collaborative works with A. de Bouard, A. Debussche, A. Deya, T. Iwabuchi, L.Thomann.

KPP systems with space-time periodic coefficients

Léo Girardin

Dans cet exposé je présenterai des extraits de résultats récents, obtenus en collaboration avec Idriss Mazari (Univ. Paris Dauphine) ou seul, qui peuvent trouver des applications par exemple en biologie ou en chimie.

Dans la première partie, on s'intéressera à des systèmes semi-linéaires de type KPP, possiblement non coopératifs mais dont le linéarisé en 0 est un opérateur parabolique coopératif à coefficients spatio-temporellement périodiques. On étudiera certaines propriétés qualitatives asymptotiques des solutions du problème de Cauchy, et comment ces propriétés sont gouvernées par la valeur propre principale du linéarisé.

Dans la deuxième partie, on s'intéressera à l'optimisation de cette valeur propre. On étudiera un problème d'optimisation spectrale naturel mais non convexe, et qui donne donc lieu à une preuve et à des résultats originaux.

Des éléments de preuve seront donnés en fonction du temps disponible.

In this presentation, I will present excerpts from recent results, obtained in collaboration with Idriss Mazari (Paris Dauphine University) or alone, which may find applications in biology or chemistry, for example.

In the first part, we will focus on semilinear KPP-type systems, which may be non-cooperative but whose linearization at 0 is a cooperative parabolic operator with spatiotemporally periodic coefficients. We will study certain qualitative asymptotic properties of the solutions to the Cauchy problem, and how these properties are governed by the principal eigenvalue of the linearization.

In the second part, we will focus on the optimization of this eigenvalue. We will study a natural but non-convex spectral optimization problem, which will lead to original proofs and results.

Elements of proofs will be given depending on the time available.

Tatsuya Hosono (Osaka Metropolitan University & Université Savoie Mont Blanc)

Title: Nonlinear Fisher information and its application to 1D critical quasilinear fully parabolic Keller–Segel system

Abstract:
In this talk, we investigate the time evolution of Fisher information, which is known as the entropy production, for nonlinear diffusion equations on bounded domains with Neumann boundary conditions, extending classical results for the linear heat equation and the porous medium equation on the whole space. In particular, we introduce an alternative formulation of one-dimensional nonlinear Fisher information that reveals its time monotonicity. As an application, the existence of global solutions to the one-dimensional critical quasilinear fully parabolic Keller–Segel system with nonlinear diffusion and nonlinear sensitivity is studied. This is based on joint work with Tomasz Cieślak (IMPAN, Poland) and Kentaro Fujie (Tohoku University, Japan).

QUADRATIC GROWTH MODEL WITH DISCONTINUITY: A LINK BETWEEN MONOSTABLE AND BISTABLE TRAVELING WAVES

WONHYUNG CHOI, JUNSIK BAE*, AND YONG-JUNG KIM

Abstract. We classify traveling waves and stationary solutions of a reaction–diffusion equation arising in population dynamics with Allee-type effects. The reaction term is given by a quadratic polynomial with a discontinuity at zero, which captures finite-time extinction for sub-threshold populations. This discontinuity induces a free boundary in the wave profile, a phenomenon that distinguishes the model from the classical logistic or Allen–Cahn equations. A complete scenario is presented that connects monostable and bistable traveling waves through the wave speed parameter, thereby providing a unified framework for their dynamics.

Keywords: reaction-diffusion equation; discontinuous nonlinearity; free-boundary; traveling wave; Allee effect

(WC) School of Computer Engineering and Applied Mathematics, Hankyong National University, 327, Jungang-ro, Anseong, 17579, Republic of Korea

Email address: whchoi@hknu.ac.kr

(JB) IRMAR - UMR 6625, CNRS, Univ Rennes, Rennes, 35000, France

Email address: altena00@gmail.com

(YK) Department of Mathematical Sciences, KAIST, 291, Daehak-ro, Yuseong-gu, Daejeon, 34141, Republic of Korea

Email address: yongkim@kaist.edu

Date: September 19, 2025.

2020 *Mathematics Subject Classification.* Primary: 35K57, 35C07 Secondary: 92D25, 92D15.

Two and three dimensional numerical solver for reaction-diffusion equations

Min-Gi Lee

Abstract

We propose a scheme to solve numerically a reaction-diffusion equation in two or three space dimensions. We consider such equations involving the divergence operator where the flux is given by

$$J = K(x)E, \quad E = -\nabla p, \quad p = M(x)u.$$

We are concerned with regularities of aforementioned quantities. Namely, we expect u is integrable, p is continuous, and crossing the interface of jump discontinuity of coefficient, tangential component of E and normal component of J are continuous. Standard FEM solver lacks the normal component continuity of J, and we illustrate an example from the Thermo-Electric device, where the commercial FEM code shows quite significant mass balance error. This phenomenon is related to the corner singularity; in a certain domain the Poisson's equation fails to have $W^{2,p}$ estimate. The scheme we propose is basically one sort of methods known as Discrete Exterior Calculus (DEC), or mimetic method, but differ from them precisely at one point: In DEC, to achive the contitutive law, the discrete Hodge dual operator is implemented, but by involving the dual mesh. In 2007, Scott Wilson, in the paper *Cochain algebra on manifolds and convergence under refinement, Topology and its Applications*, showed that the inverse of discrete Hode dual operator is well-defined without dual mesh. We explore what are possible designs if we combine DEC and inverse Hodge dual operator.

Optimization problem with bang-bang property in reaction-diffusion equations with sign-change population model

Kentaro NAGAHARA (Institute of Science Tokyo)*

Abstract

This talk addresses the monostable reaction-diffusion logistic equation related to mathematical biology. This equation, also known as the Fisher-KPP equation, was proposed by J.G. Skellam in 1951 as a model equation describing population dynamics of organisms, and it has been extensively studied. In this talk, we consider a reaction-diffusion logstic model where the intrinsic growth rate is sign-changing. We examine the linearized eigenvalue problem around the trivial solution and consider resource distribution that maximizes the survival chance of a biological population. We show that the maximizer of the eigenvalue are always bang-bang type, independent of the network structure. This behavior also differs from the global maximum solution for the total population size.

This work was supported by JSPS KAKENHI Grant Number 24K16750.
Keywords: Optimization problem, Generalized eigenvalue problem, Total Poopulation.
* e-mail: nagahara@ila.isct.ac.jp
web: https://www.nk.ila.titech.ac.jp/

Connections between traveling wave and entire solutions in reaction-diffusion equations

Hirokazu Ninomiya

Abstract

In this talk, we consider propagation phenomena in bistable reaction-diffusion equations and compare the relationships between traveling wave solutions and entire solutions based on previous studies. I will explain how to construct entire solutions by combining traveling wave solutions, as well as studies that provide insights into the existence of traveling waves through the analysis of entire solutions. In addition, I will present a new method to construct entire solutions in an $(N-1)$-dimensional space from traveling wave solutions in an N-dimensional space. This talk is partially based on joint work with Masaharu Taniguchi (Okayama University).

References

[1] H. Guo, and K. Wang. Some new bistable transition fronts with changing shape, *Mathematische Annalen* **392** (2025): 3797-3850.

[2] H. Ninomiya. Entire solutions and traveling wave solutions of the Allen-Cahn-Nagumo equation. *Discrete and Continuous Dynamical Systems: Series A* **39.4** (2019): 2001-2019.

[3] H. Ninomiya. Entire solutions of the Allen-Cahn-Nagumo equation in a multi-dimensional space. *Discrete and Continuous Dynamical Systems* **41.1** (2021): 395-412.

[4] H. Ninomiya and M. Taniguchi. Traveling Front Solutions of Dimension n Generate Entire Solutions of Dimension (n-1) in Reaction-Diffusion Equations as the Speeds Go to Infinity. *Archive for Rational Mechanics and Analysis* **249.1** (2025): Article Number 13.

Regularization by noise for some modulated dispersive PDEs

Tristan Robert
Université de Lorraine

Abstract

In this talk, we will consider nonlinear dispersive PDEs where a deterministic noise is added as a distributional time coefficient in front of the dispersion. Despite the roughness of the noise term, we will see that any semilinear dispersive PDE with this noise term is well-posed at least in the same range of regularity as its noiseless counterpart, as soon as well-posedness relies on linear space-time estimates. Building on previous works on this model, we will also observe several regularization by noise phenomena provided that the noise is irregular enough : large data global well-posedness for focusing mass-critical equations, well-posedness at super-critical regularity for strongly non-resonant equations through improved multilinear estimates, and improvement on the Cauchy theory for Kadomtsev-Petviashvili equations through short-time multilinear estimates on longer time scales.

Title:
Reaction-diffusion approximation of nonlocal interactions in high dimensions

Speaker:
Yoshitaro Tanaka (Future University Hakodate),
Hiroshi Ishii (Hokkaido University)

Abstract:
Motivated by pattern formations, many evolution equations incorporating spatial convolution with suitable integral kernel have been proposed. Numerical simulations of these nonlocal evolution equation can reproduce various patterns depending on the kernel shape. In this talk, we consider the relationship between these nonlocal evolution equations and a reaction-diffusion system. By controlling parameters and taking a singular limit in the reaction-diffusion system, we show that nonlocal interactions that satisfy dimensional conditions can be approximated by the reaction-diffusion systems in general. This research is a collaboration with Hiroshi Ishii of Hokkaido University.

Laurent Thomann (Université de Lorraine)

Renormalization of a quadratic Schrödinger model with additive noise

The study is devoted to the interpretation and wellposedness of a quadratic stochastic NLS model with an additive space-time fractional noise. We will focus on the case where the noise is rough, and we exhibit an explicit Bourgain-Wick renormalization procedure allowing to restore the convergence of some approximated solutions. This a joint work with Aurélien Deya (CNRS & Université de Lorraine) and Reika Fukuizumi (Waseda University).

Domination criterion for some positive operators and quasi-stationary distributions

Denis Villemonais*†

After a short introduction to the concept of quasi-stationary distributions, I will present the typical and well known "finite state space" convergence results. In a second time, I will present domination criteria for the quasi-compactness of positive operator and show some applications of these spectral theoretical results for the study of quasi-stationary distributions. The talk will conclude with an illustration on the interplay between these results and recent ones on weighted branching processes, obtained in collaboration with Nicolas Zalduendo.

Preprint: `https://arxiv.org/abs/2510.19573`

*IRMA, Université de Starsbourg, France
†Institut Universiatire de France
denis.villemonais@unistra.fr